On cohomologies of some ordinary p-adic Galois representations

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Abstract

Let G be the absolute Galois group of a p-adic field. A p-adic representation of G is said to be *ordinary* if it has a decreasing, exhaustive and separated filtration of G-stable subspaces such that the inertia subgroup of G acts by a power of the p-adic cyclotomic character at each graded piece. In this talk, we look at some examples of ordinary representations and prove the vanishing of certain Galois cohomology groups with coefficients in such representations. In particular, we give a necessary and sufficient condition for the vanishing to hold in the case given by an abelian variety with good ordinary reduction. Using these local results, we give some consequences for Galois cohomology of global Galois representations associated with abelian varieties. This gives a generalization of a result due to Coates, Sujatha and Wintenberger for these cases.

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1 Introduction

The vanishing of cohomology groups associated with p-adic Galois representations defined by elliptic curves is one of the useful results towards generalization of methods in Iwasawa theory to larger Galois extensions. Such vanishing enables the computation of Euler characteristics for discrete modules associated to p-adic Galois representations [4] and Selmer groups of elliptic curves over extensions containing all p-power roots of unity [2], [3], [15].

The aim of this note is to discuss the the vanishing of cohomology groups with values in an ordinary p-adic Galois representation with respect to some Galois extensions which contain all the p-power roots of unity.

Let p be a prime number and K a finite extension of $\mathbb{Q}_p$. Fix a separable closure $\overline{K}$ of K, write $G_K := \mathrm{Gal}(\overline{K}/K)$ for the absolute Galois group of K. Let V be a finite-dimensional representation of G_K over $\mathbb{Q}_p$. We denote by

$$\rho : G_K \longrightarrow \mathrm{GL}(V)$$

the homomorphism giving the action of G_K on the vector space V.

For a general finite-dimensional vector space V over $\mathbb{Q}_p$ and a compact subgroup G of $\mathrm{End}(V)$, we write $H^m(G, V)$ $(m = 0, 1, \ldots)$ for the cohomology groups of G acting on V defined by continuous cochains, where V is endowed with the p-adic topology.

Definition 1.1. The vector space V has *vanishing G-cohomology* if the cohomology groups $H^m(G, V)$ are trivial for all $m \geq 0$.

Let $K(\mu_{p^\infty})$ be the smallest field extension of K (in $\overline{K}$) which contains all the roots of unity of order a power of p. Let $G_{K(\mu_{p^\infty})}$ be the subgroup of G_K that corresponds to $K(\mu_{p^\infty})$. Denote by G_V (resp. H_V) the image of G_K (resp. $G_{K(\mu_{p^\infty})}$) under ρ. The group

G_V (resp. H_V) can be realised as the Galois group of $K(V)$ over K (resp. $K(\mu_{p^\infty})$). In their study of Euler characteristics of p-adic Galois representations, Coates, Sujatha and Wintenberger proved the following result:

Proposition 1.2 ([4, Propositions 4.1 and 4.2])**.** *Let (ρ, V) be a potentially crystalline p-adic representation of G_K. Let K' be a finite extension of K such that $(\rho|_{G_{K'}}, V|_{G_{K'}})$ is crystalline. Let Φ be the endomorphism acting on the filtered φ-module $D(V|_{G_{K'}})$ associated with V. Assume that*
(1) *the eigenvalues of Φ are q-Weil numbers of weight w and*
(2) *the determinant of Φ is a rational number.*
Then:
(i) *If $w \neq 0$, then V has vanishing G_V-cohomology.*
(ii) *If w is odd, then V has vanishing H_V-cohomology.*

Let X be a proper smooth variety over K. For an integer i, let $H^i_{\text{ét}}(X_{\overline{K}}, \mathbb{Q}_p)$ be the ith étale cohomology group of X. This is a p-adic representation of G_K of finite-dimension over $\mathbb{Q}_p$. Then we have the following

Corollary 1.3. *Let X be a proper smooth variety over K with potential good reduction. Let $V = H^i_{\acute{e}t}(X_{\overline{K}}, \mathbb{Q}_p)$ and ρ the continuous homomorphism giving the action of G_K on V. Then*:
(i) *If $i \neq 0$, then V has vanishing G_V-cohomology.*
(ii) *If i is odd, then V has vanishing H_V-cohomology.*

We are interested in identifying the Galois extensions L of K with respect to which the corresponding cohomology groups vanish as in Theorem 1.2. More precisely, if (ρ, V) is a p-adic Galois representation as above and L is a subextension of $\overline{K}$, we want to know when does V have vanishing J_V-cohomology, where $J_V = \rho(G_L)$. In the case where $V = H^1_{\text{ét}}(E_{\overline{K}}, \mathbb{Q}_p)$ is given by an elliptic curve E over K, it can be shown that when L is given by the field of p-power division points of another elliptic curve E' over K, then V has vanishing J_V-cohomology depending on the reduction types of E and E' (see §3). For instance, if E and E' both have good ordinary reduction over K, then V does not have vanishing J_V-cohomology. The theorem that we will discuss provides a necessary and sufficient condition on L in order for some ordinary representation V to have vanishing J_V-cohomology. This generalizes our observation in the case of elliptic curves with good ordinary reduction.

2 Notations and some definitions

Let p be a prime number. Throughout this note K denotes a p-adic local field, that is a finite extension of the field $\mathbb{Q}_p$ of p-adic numbers. We denote its ring of integers by $\mathcal{O}_K$ and its residue field by k. We fix a separable closure $\overline{K}$ of K. For a subextension L of $\overline{K}$, we write $G_L := \mathrm{Gal}(\overline{K}/L)$. Let K^{nr} be the maximal unramified extension of K in $\overline{K}$. We put $I_K := \mathrm{Gal}(\overline{K}/K^{\mathrm{nr}})$, the inertia subgroup of G_K.

A p-adic Galois representation of G_K is denoted by (ρ, V), where V is a finite-dimensional $\mathbb{Q}_p$-vector space and ρ denotes the continuous homomorphism giving the action of G_K on V. For such a p-adic Galois representation, we denote by $K(V)$ the fixed subfield of $\overline{K}$ by the kernel of ρ. Let L be a subfield of $\overline{K}$ which contains K. As G_L is a profinite group, the image $\rho(G_L)$ of G_L under ρ is a compact p-adic Lie group contained in $\mathrm{GL}_n(\mathbb{Z}_p)$, where $n = \dim V$. We may identify the group $\rho(G_L)$ with the Galois group $\mathrm{Gal}(K(V)/K(V) \cap L)$.

For a positive integer m, let μ_{p^m} denote the group of p^mth roots of unity. We denote by μ_{p^∞} the union of all μ_{p^m} as m runs over the set of all positive integers. We let $K(\mu_{p^\infty}) =$

$\bigcup_m K(\mu_{p^m})$ denote the field extension obtained by adjoining to K all the p-power roots of unity. The character $\chi : G_K \to \mathbb{Z}_p^\times$ always denotes the p-adic cyclotomic character (that is, the character of G_K such that $g(\zeta) = \zeta^{\chi(g)}$ if $g \in G_K$ and $\zeta^{p^m} = 1$ for some m). The fixed subfield of $\overline{K}$ by the kernel of χ is $K(\mu_{p^\infty})$.

Let A be a g-dimensional abelian variety over K. The Tate-module $T_p(A) := \varprojlim A_{p^n}(\overline{K})$ of A is a free $\mathbb{Z}_p$-module of rank $2g$. Set $V_p(A) = T_p(A) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$. Then $V_p(A)$ is a $2g$-dimensional p-adic Galois representation of G_K which is dual to $H^1_{\text{ét}}(A_{\overline{K}}, \mathbb{Q}_p)$. In this case, we write $K(A_\infty)$ instead of $K(V_p(A))$.

Definition 2.1. Let F be a field. For an algebraic extension F' of F, we say that F' is a *prime-to-p extension* of F if F' is a union of finite extensions over F of degree prime-to-p. If F' is a prime-to-p extension over some finite extension field of F, we say that F' is a *potential prime-to-p extension* of F.

Clearly, if F' is a potential prime-to-p extension of F, then every intermediate field F'' (with $F \subseteq F'' \subseteq F'$) is a potential prime-to-p extension of F. In this note, we will consider algebraic extensions L of a p-adic field K whose residue field k_L is a potential prime-to-p extension over the residue field k of K. Here are some examples of such fields:

Example 2.2. (1) $L = K(\mu_{p^\infty})$.
(2) More generally, if L is a subfield of

$$K((K^\times)^{p^{-\infty}}) := \bigcup_{m=1}^{\infty} K(x^{p^{-m}} : x \in K^\times),$$

then k_L is a finite extension, and thus a potential prime-to-p extension of k (cf. [9, Lemma 2.3]).
(3) Let (ρ, V) be a p-adic Galois representation of G_K. If $\rho(I_K)$ is an open subgroup of $\rho(G_K)$, then the residue field k_L of $L = K(V)$ is potential prime-to-p over k.

3 Related Results

Theorem 3.1 ([5, Theorem 4.8])**.** *Let X be a proper smooth variety over K with potential good ordinary reduction (in the sense of Bloch-Kato [1]) and $i > 0$ an odd integer. Consider an elliptic curve E/K with potential good supersingular reduction.*
(a) Let $V = H^i_{ét}(X_{\overline{K}}, \mathbb{Q}_p)$ and $L = K(E_\infty)$. Then V has vanishing J_V-cohomology, where $J_V = \rho(G_L)$;
(b) Let $V' = V_p(E)$ and $L' = K(V)$, where V is the $\mathbb{Q}_p$-vector space in (a). Then V' has vanishing $J_{V'}$-cohomology, where $J_{V'} = \rho_E(G_{L'})$.

Suppose E is an elliptic curve with potential good reduction over K and assume $L = K(E'_\infty)$ is given by another elliptic curve E'. By distinguishing the reduction types of E and E', we obtain the following result on the vanishing of J_V-cohomology of $V = V_p(E)$, where $J_V = \rho_E(G_L)$. It also involves the case where E' has potential multiplicative reduction. This extends some of the results obtained in [11].

Theorem 3.2 ([5, Theorems 5.2 and 5.6])**.** *Let E, E' and J_V be as given above. The vanishing of J_V-cohomology of $V = V_p(E)$ is given by the following table*:

<table>
<tr><th>E</th><th>E'</th><th>J_V-cohomology vanish</th></tr>
<tr><td rowspan="3">ordinary</td><td>ordinary</td><td>No</td></tr>
<tr><td>supersingular</td><td>Yes</td></tr>
<tr><td>multiplicative</td><td>Yes</td></tr>
<tr><td rowspan="4">supersingular with FCM</td><td>ordinary</td><td>Yes</td></tr>
<tr><td>supersingular with FCM</td><td>Yes*</td></tr>
<tr><td>supersingular without FCM</td><td>Yes</td></tr>
<tr><td>multiplicative</td><td>Yes</td></tr>
<tr><td rowspan="4">supersingular without FCM</td><td>ordinary</td><td>Yes</td></tr>
<tr><td>supersingular with FCM</td><td>Yes</td></tr>
<tr><td>supersingular without FCM</td><td>Yes*</td></tr>
<tr><td>multiplicative</td><td>Yes</td></tr>
</table>

The symbol $*$ means that the vanishing in such cases hold under the additional assumption that the group $E(L')[p^\infty]$ of L'-rational points of E of p-power order is finite for every finite extension L' of L. For these cases the vanishing of all cohomology groups is determined by the vanishing of the group $H^0(J_V, V)$.

For an abelian variety A over K, we denote by $K(A[p])$ the field extension obtained by adjoining to K the coordinates of points of $A(\overline{K})$ of order p. In [11], Ozeki studied the finiteness of the torsion subgroup of $A(L)$ where L is an algebraic extension of K which contains the field $K(\mu_{p^\infty})$. His results generalize a theorem of Imai [8] for abelian varieties. There is a further generalization of this in [9]. For abelian varieties with good ordinary reduction, Ozeki proved the following

Theorem 3.3 ([11, Corollary 2.1]). *Let A be an abelian variety over K which has good ordinary reduction. Let L be a Galois extension of K with residue field k_L. Assume that L contains $K(\mu_{p^\infty})$ and $K(A[p])$. Then the following statements are equivalent:*
(1) $H^0(G_L, V_p(A)) = 0$;
(2) $A(L)[p^\infty]$ *is finite*;
(3) k_L *is a potential prime-to-p extension over k.*

4 Ordinary Representations

Definition 4.1. A p-adic Galois representation (ρ, V) of G_K is said to be *ordinary* if there exists a filtration by G_K-invariant subspaces $\{\mathrm{Fil}^i V\}_{i\in\mathbb{Z}}$ with the following properties:

$$\begin{aligned} \mathrm{Fil}^{i+1} V \subseteq \mathrm{Fil}^i V \text{ for all } i, \\ \mathrm{Fil}^i V = V \text{ for } i \ll 0 \text{ and} \\ \mathrm{Fil}^i V = 0 \text{ for } i \gg 0, \end{aligned}$$

such that the inertia subgroup I_K acts on the ith graded quotient $\mathrm{Fil}^i V/\mathrm{Fil}^{i+1} V$ by the ith power of the p-adic cyclotomic character.

Example 4.2. (1) Let A be an abelian variety over K with good ordinary reduction. Let $\mathcal{A}$ denote the Néron model of A over $\mathcal{O}_K$. Denote by $\mathcal{A}(p)$ the p-divisible group associated with $\mathcal{A}$ and we denote its connected component and maximal étale quotient by $\mathcal{A}(p)^0$ and $\mathcal{A}(p)^{\text{ét}}$, respectively. For $G = \mathcal{A}(p), \mathcal{A}(p)^0$, and $\mathcal{A}(p)^{\text{ét}}$, let $T_p(G)$ be the Tate-module and $V_p(G) = T_p(G) \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$. The connected-étale sequence induces the exact sequence of G_K-modules

$$0 \to V_p(\mathcal{A}(p)^0) \to V_p(\mathcal{A}(p)) \to V_p(\mathcal{A}(p)^{\text{ét}}) \to 0.$$

The inertia subgroup I_K acts on $V_p(\mathcal{A}(p)^0)$ by χ, while $V_p(\mathcal{A}(p)^{\text{ét}})$ is unramified.
(2) Let $f_{12}(z) = \sum_{n=1}^{\infty} \tau(n) e^{2\pi i n z}$ be the normalized cusp form of weight 12, level 1. Deligne constructed a 2-dimensional p-adic representation $V_p(f_{12})$ of G_K associated to f_{12}. Mazur and Wiles proved that, under a suitable condition, there exists a filtration on $V_p(f_{12})$ on which the action of I_K has occurring characters χ^0 and χ^{11}.

By a result of Perrin-Riou, we have the following characterization of ordinary representations:

Theorem 4.3 ([12, Theorem 1.5])**.** *Let V be an ordinary representation. Then V is a semi-stable representation.*

Suppose V is ordinary. In view of this theorem, we associate to V a filtered (φ, N)-module $D(V)$. Let $q = p^f$ be the cardinality of the residue field k of K. Then the fth iterate $\Phi = \varphi^f$ of φ is a K_0-linear endomorphism of $D(V)$, where K_0 is the maximal unramified extension of $\mathbb{Q}_p$ contained in K.

Definition 4.4. A *q-Weil number of weight* $w \geq 0$ is an algebraic integer α satisfying $|\iota(\alpha)| = q^{w/2}$ for all field embeddings $\iota : \mathbb{Q}(\alpha) \hookrightarrow \mathbb{C}$.

Definition 4.5. A (φ, N)-module $D(V)$ over K_0 is said to be *pure of weight* w if the characteristic polynomial $\det(X - \Phi)$ has coefficients in $\mathbb{Q}$ and all the roots are q-Weil numbers of weight w.

It is known that if w is an integer and V is a semistable representation of G_K such that (φ, N)-module $D(V)$ over K_0 is pure of weight w, then V is crystalline (see for instance Remark 4.3 in [4] for a proof of this fact). Thus we have the following

Corollary 4.6. *Let V be an ordinary representation. Assume that the associated (φ, N)-module $D(V)$ is pure of integer weight. Then V is a crystalline representation.*

From this corollary, we obtain the following special case of Proposition 1.2:

Proposition 4.7. *Let V be an ordinary representation. Assume that the associated (φ, N)-module $D(V)$ is pure of odd weight. Suppose further that the determinant of the endomorphism Φ of $D(V)$ is a rational number. Then V has vanishing G_V-cohomology and vanishing H_V-cohomology.*

5 The main result

Let (ρ, V) be a finite-dimensional representation of G_K. We consider the following condition:

Assumption 5.1. V has a filtration of length two

$$0 = V_{-1} \subsetneq V_0 \subsetneq V_1 = V,$$

such that I_K acts on V_1/V_0 (resp. V_0) by χ^a (resp. χ^b), where a and b are distinct integers.

Theorem 5.2. *Let (ρ, V) be a finite-dimensional potentially crystalline representation of G_K. Let K' be a finite extension of K such that $(\rho_{G_{K'}}, V)$ is crystalline and satisfies Assumption 5.1. Let $\Phi = \varphi^f$ denote the endomorphism acting on the associated filtered φ-module $D(V)$ Let L be a Galois extension of K containing $K(\mu_{p^\infty})$. Assume that*
(1) *$D(V)$ is pure of odd weight,*
(2) $\det \Phi \in \mathbb{Q}$,
(3) *the residue fielf k_L of L is a potential prime-to-p extension of K,*
(4) *$V^{G_{L'}} = 0$ for every finite extension L' of L.*
Put $J_V = \rho(G_L)$. Then V has vanishing J_V-cohomology.

Combining this result with Theorem 3.3 gives the following

Corollary 5.3. *Let A be an abelian variety over K with potential good ordinary reduction. Let L/K be a p-adic Lie extension containing $K(\mu_{p^\infty})$ and $K(A[p])$. Put $V = V_p(A)$ and we write $J_V = \rho(G_L)$. The following conditions are equivalent:*
(1) *k_L is a potential prime-to-p extension of k;*
(2) *$V^{G_L} = 0$;*
(3) *V has vanishing J_V-cohomology.*

6 Proof of Theorem 5.2

Assumptions (1) and (2) imply that the field $K(\mu_{p^\infty})$ is a finite extension of $K(V) \cap K(\mu_{p^\infty})$. So replacing K by a finite extension, we may assume that $K(V)$ contains $K(\mu_{p^\infty})$. Put $I_V = \rho(I_K)$ and $J_V = \rho(G_L)$. Let $N = K(V)$, $N_\infty = N(\mu_{p^\infty})$, and $M = K(V) \cap L$. We make the following identification of p-adic Lie groups

$$\begin{aligned} I_V &\simeq \mathrm{Gal}(K(V)/N), \\ H_V &\simeq \mathrm{Gal}(K(V)/K(\mu_{p^\infty})), \\ J_V &\simeq \mathrm{Gal}(K(V)/M). \end{aligned}$$

We also let $M' = L \cap N_\infty = M \cap N_\infty$, $K_0 = L \cap N = M \cap N$, and put $G = \mathrm{Gal}(M/K_0)$, $H = \mathrm{Gal}(M/M')$ and $Y = G/H$. Then we have the following diagram of fields:

$$\begin{tikzcd}
K(V) \arrow[d, dash] \arrow[dr, dash] \arrow[dd, dash, bend right=60, "I_V"'] & & L \arrow[dl, dash] \\
N_\infty \arrow[d, dash] \arrow[dr, dash] & M \arrow[d, dash] \arrow[d, dash, bend left=50, "H"] \arrow[dd, dash, bend left=80, "G"] & \\
N \arrow[dr, dash] & M' \arrow[d, dash] \arrow[d, dash, bend left=50, "Y"] & \\
& K_0 \arrow[d, dash] & \\
& K &
\end{tikzcd}$$

By assumption (3), the residue field k_M of M is a potential prime-to-p extension of k. Since N/K is unramified, the the field K_0 is of finite degree over K. Replace K with K_0, we may assume that $G = \mathrm{Gal}(M/K)$. Then we have the following commutative diagram with exact rows and surjective vertical maps

$$\begin{tikzcd}
1 \arrow[r] & \mathrm{Gal}(K(V)/N_\infty) \arrow[r] \arrow[d, two heads] & I_V \arrow[r] \arrow[d, two heads] & \mathrm{Gal}(N_\infty/N) \arrow[r] \arrow[d, two heads] & 1 \\
1 \arrow[r] & H \arrow[r] & G \arrow[r] & Y \arrow[r] & 1
\end{tikzcd} \tag{1}$$

Moreover, the above diagram is compatible with the action by inner automorphisms.

We now make use of assumption 5.1. Let $0 = V_{-1} \subsetneq V_0 \subsetneq V_1 = V$ be a filtration on V which satisfies the additional conditions given in assumption 5.1. Let $n = \dim_{\mathbb{Q}_p} V$ and

$d = \dim_{\mathbb{Q}_p} V_0$. Let $\{v_1, \ldots, v_n\}$ be a basis of V such that $\{v_1, \ldots, v_d\}$ is a basis of V_0. Then the representation ρ of G_V on V can be written by

$$\begin{pmatrix} (\chi^a)^{\oplus d} \cdot U_1(\sigma) & T(\sigma) \\ 0 & (\chi^b)^{\oplus n-d} \cdot U_2(\sigma) \end{pmatrix} \qquad \sigma \in G_V, \tag{2}$$

where $U_1 : G_V \to \mathrm{GL}_d(\mathbb{Z}_p)$ and $U_2 : G_V \to \mathrm{GL}_{n-d}(\mathbb{Z}_p)$ are continuous unramified homomorphisms and $T : G_V \to M_{d\times n-d}(\mathbb{Z}_p)$ is a continuous map. Here, for integers r and s, $(\chi^r)^{\oplus s}(\sigma)$ denotes the $s \times s$ diagonal matrix with entries $\chi^r(\sigma)$.

The commutative diagram (1) and the representation (2) together imply that the action of the group G on H_V/J_V is given by

$$\sigma \cdot \tau \cdot \sigma^{-1} = \tau^{\varepsilon(\sigma)}, \qquad \sigma \in G, \quad \tau \in H_V/J_V,$$

where ε denotes the character χ^{a-b}. This is non-trivial and is continuous with open image in $\mathbb{Z}_p^\times$.

We use the following lemma:

Lemma 6.1. *Let $\psi : G \to \mathrm{GL}_{\mathbb{Q}_p}(W)$ be a continuous $\mathbb{Q}_p$-linear representation of G on a finite-dimensional $\mathbb{Q}_p$-vector space W. Let $\varepsilon : G \to \mathbb{Z}_p^\times$ be a continuous character of G whose image is open in $\mathbb{Z}_p^\times$. Assume that the following relation holds:*

$$\sigma \cdot \tau \cdot \sigma^{-1} = \tau^{\varepsilon(\sigma)}$$

for all $\sigma \in G$, $\tau \in H$. Then after a finite extension K'/K, the subgroup H acts unipotently on W.

Considering $H^n(J_V, V)$ as a representation of G, Lemma 6.1 implies that there is a subgroup $\Omega \subseteq H_V/J_V$ of finite index such that Ω acts unipotently on $H^n(J_V, V)$. Replacing K by a finite extension, we may identify Ω with H_V/J_V.

Lemma 6.2. *Let $\varphi : \mathscr{U} \to \mathrm{GL}_{\mathbb{Q}_p}(W)$ be a representation of a group $\mathscr{U}$ on a finite-dimensional $\mathbb{Q}_p$-vector space W. Suppose $\mathscr{U}$ acts unipotently on W. Then $W^{\mathscr{U}} = 0$ if and only if $W = 0$.*

Now, conditions (i) and (ii) imply that V has vanishing H_V-cohomology. We now prove that V has vanishing J_V-cohomology by induction. First, we have

$$H^0(\Omega, H^0(J_V, V)) \simeq H^0(H_V, V) = 0.$$

Since Ω acts unipotently on $H^0(J_V, V)$, Lemma 6.2 implies that $H^0(J_V, V)$. Assume that $H^r(J_V, V)$ for $r = 0, \ldots, m-1$, where $m \geq 1$. The Hochschild-Serre spectral sequence gives an exact sequence

$$H^m(H_V, V) \to H^0(\Omega, H^m(J_V, V)) \to H^{m+1}(\Omega, H^0(J_V, V)).$$

As the first and third terms are trivial we have $H^0(\Omega, H^m(J_V, V)) = 0$. Once again since Ω acts unipotently on $H^m(J_V, V)$, applying Lemma 6.2 implies that V has vanishing J_V-cohomology. This ends the proof of our main theorem.

References

[1] S. Bloch and K. Kato, *p-adic étale cohomology*, Inst. Hautes Etudes Sci. Publ. Math. **63**, (1986), 107–152.

[2] J. Coates and S. Howson, *Euler Characteristics and elliptic curves II*, J. Math. Soc. Japan **53** (2001), 175–235.

[3] J. Coates, P. Schneider and R. Sujatha, *Links between cyclotomic and GL2 Iwasawa theory*, In: Kazuya Kato's fiftieth birthday, 187–215, Doc. Math., 2003, Extra Vol. (electronic).

[4] J. Coates, R. Sujatha and J.-P. Wintenberger, *On Euler-Poincaré characteristics of finite dimensional p-adic Galois representations*, Inst. Hautes Etudes Sci. Publ. Math. **93**, (2001), 107–143.

[5] J. Dimabayao, *On the vanishing of cohomologies of p-adic Galois representations associated with elliptic curves*, preprint, 2014, arXiv:1309.7240.

[6] J. D. Dixon, M. P. F. du Sautoy, A. Mann and D. Segal, Analytic Pro-p Groups, Second edition, Cambridge Studies in Advanced Mathematics, 61. Cambridge University Press, Cambridge, 1999.

[7] L. Illusie, *Réduction semi-stable ordinaire, cohomologie étale p-adique et Cohomologie de de Rham, d'apres Bloch-Kato et Hyodo*, In: Périodes p-adiques (Bures-sur-Yvette, France, 1988), 209–220, Astérisque, 223, Soc. Math. France, Montrouge, 1994.

[8] H. Imai, *A remark on the rational points of abelian varieties with values in cyclotomic* $\mathbb{Z}_p$*-extensions*, Proc. Japan Acad. **51** (1975), 12–16.

[9] Y. Kubo and Y. Taguchi, *A generalization of a theorem of Imai and its applications to Iwasawa theory*, Math. Z. **275** (2013), 1181–1195.

[10] M. Lazard, *Groupes analytiques p-adiques*, Inst. Hautes Etudes Sci. Publ. Math. **26** (1965), 389–603.

[11] Y. Ozeki, *Torsion points of abelian varieties with values in infinite extensions over a p-adic field*, Publ. Res. Inst. Math. Sci. **45** (2009), 1011–1031.

[12] B. Perrin-Riou, *Représentations p-adiques ordinaires* In: Périodes p-adiques (Bures-sur-Yvette, France, 1988), 185–208, Astérisque, **223**, Soc. Math. France, Montrouge, 1994.

[13] J.-P. Serre, *Propriétés galoisiennes des points d'ordre fini des courbes elliptiques*, Invent. Math. **15** (1972), 259–331.

[14] J.-P. Serre, Abelian l-adic representations and elliptic curves, With the collaboration of Willem Kuyk and John Labute. Second Edition, Advanced Book Classics. Addison-Wesley Publishing Company, Advanced Book Program, Redwood City, CA, 1989.

[15] S. L. Zerbes, *Generalised Euler characteristics of Selmer groups*, Proc. Lond. Math. Soc. (3) **98** (2009), 775–796.